LIBERTY PAPER SET

STD. 12 : Mathematics

Full Solution

Time : 3 Hours **ASSIGNMENT PAPER 1**

PART A

1. (C) **2.** (B) **3.** (C) **4.** (B) **5.** (A) **6.** (A) **7.** (D) **8.** (B) **9.** (C) **10.** (A) **11.** (B) **12.** (D) **13.** (C) **14.** (A) **15.** (B) **16.** (D) **17.** (C) **18.** (A) **19.** (B) **20.** (C) **21.** (A) **22.** (B) **23.** (C) **24.** (C) **25.** (D) **26.** (A) **27.** (D) **28.** (C) **29.** (B) **30.** (D) **31.** (C) **32.** (B) **33.** (A) **34.** (B) **35.** (C) **36.** (A) **37.** (D) **38.** (A) **39.** (C) **40.** (B) **41.** (C) **42.** (A) **43.** (D) **44.** (A) **45.** (C) **46.** (A) **47.** (B) **48.** (C) **49.** (A) **50.** (D)

PART B

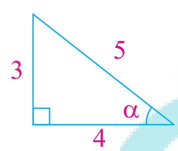
SECTION A

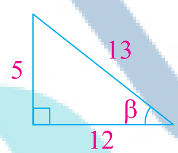
1.

L.H.S. $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

Take, $\cos^{-1}\frac{4}{5} = \alpha$, $\cos^{-1}\frac{12}{13} = \beta$

$\therefore \quad \cos\alpha = \frac{4}{5}$, $\cos\beta = \frac{12}{13}$





$\therefore \quad \sin\alpha = \frac{3}{5}$, $\sin\beta = \frac{5}{13}$

Here, $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$= \left(\frac{4}{5}\times\frac{12}{13}\right) - \left(\frac{3}{5}\times\frac{5}{13}\right)$

$= \frac{48}{65} - \frac{15}{65}$

$\cos(\alpha+\beta) = \frac{33}{65}$

$\therefore \quad \alpha+\beta = \cos^{-1}\left(\frac{33}{65}\right)$

$\therefore \quad \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

2.

L.H.S. $= \sin^{-1}(2x\sqrt{1-x^2})$

Suppose, $x = \sin\theta$,

$\therefore\ \theta = \sin^{-1}x,\ \theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$= \sin^{-1}(2\sin\theta\sqrt{1-\sin^2\theta})$

$= \sin^{-1}(2\sin\theta\cos\theta)$

$= \sin^{-1}(\sin 2\theta)$

Here, $-\frac{1}{\sqrt{2}} \le x \le \frac{1}{\sqrt{2}}$

$\therefore\ -\sin\frac{\pi}{4} \le \sin\theta \le \sin\frac{\pi}{4}$

$\therefore\ \sin\left(-\frac{\pi}{4}\right) \le \sin\theta \le \sin\frac{\pi}{4}$

$\therefore\ -\frac{\pi}{4} \le \theta \le \frac{\pi}{4}$

$\therefore\ -\frac{\pi}{2} \le 2\theta \le \frac{\pi}{2}$

$\therefore\ 2\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ .... (1)

$\therefore\ \sin^{-1}(\sin 2\theta) = 2\theta$ ($\because$ From equation (1))

$= 2\sin^{-1}x$

= R.H.S.

3.

Suppose, $u = x^y$ and $v = y^x$

$\therefore\ u + v = 1$

Now, differentiate w.r.t. x,

$\frac{du}{dx} + \frac{dv}{dx} = 0$ ...... (1)

Here, $u = x^y$

Take both the sides *log*,

$\log u = y\log x$

Now, differentiate w.r.t. x,

$$\frac{du}{dx}\frac{1}{u} = y\frac{d}{dx}\log x + \log x\frac{d}{dx}y$$

$$\therefore \quad \frac{du}{dx}\frac{1}{u} = y\cdot\frac{1}{x} + \log x\frac{dy}{dx}$$

$$\therefore \quad \frac{du}{dx} = u\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$

$$= x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right]$$

$$\therefore \quad \frac{du}{dx} = x^{y-1}y + x^y\log\frac{dy}{dx} \quad ......\ (2)$$

Now, $v = y^x$

Take both the side *log*,

$\log v = x\log y$

Now, differentiate w.r.t. x,

$$\frac{1}{v}\frac{dv}{dx} = x\frac{d}{dx}\log y + \log y\frac{d}{dx}x$$

$$\therefore \quad \frac{1}{v}\frac{dv}{dx} = x\cdot\frac{1}{y}\frac{dy}{dx} + \log y$$

$$\therefore \quad \frac{dv}{dx} = v\left[\frac{x}{y}\frac{dy}{dx} + \log y\right]$$

$$= y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \quad ......\ (3)$$

Put the value of equation (2) and (3) in equation (1),

$$x^{y-1}y + x^y\log x\frac{dy}{dx} + y^{x-1}x\frac{dy}{dx} + y^x\log y = 0$$

$$\frac{dy}{dx}\left[x^y\log x + y^{x-1}x\right] = -y^x\log y - x^{y-1}y$$

$$\frac{dy}{dx} = -\frac{[y^x\log y + x^{y-1}y]}{[x^y\log x + y^{x-1}x]}$$

4.

➪ $$I = \int\frac{dx}{\sqrt{8+3x-x^2}}$$

$$= \int\frac{dx}{\sqrt{-(x^2-3x-8)}}$$

$$= \int\frac{dx}{\sqrt{-\left(x^2-2\left(\frac{3x}{2}\right)+\frac{9}{4}-\frac{9}{4}-8\right)}}$$

$$= \int\frac{dx}{\sqrt{-\left(\left(x-\frac{3}{2}\right)^2-\frac{41}{4}\right)}}$$

$$= \int\frac{dx}{\sqrt{\left(\frac{\sqrt{41}}{2}\right)^2-\left(x-\frac{3}{2}\right)^2}}$$

$$= \sin^{-1}\left(\frac{x-\frac{3}{2}}{\frac{\sqrt{41}}{2}}\right) + c$$

$$\therefore \quad I = \sin^{-1}\left(\frac{2x-3}{\sqrt{41}}\right) + c$$

5.

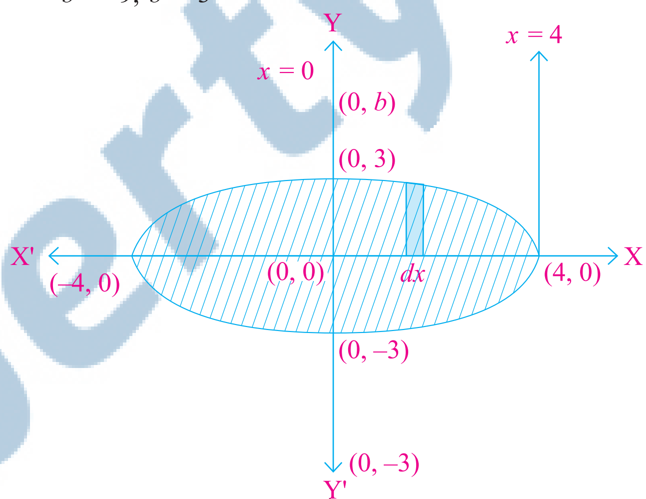
➪ $9x^2 + 16y^2 = 144$

$$\therefore \quad \frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$a^2 = 16,\ a = 4\ (a > b)$

$b^2 = 9,\ b = 3$



Required Area :

A = 4 × Area bounded in the first quadrant

∴ A = 4|I|

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\therefore \quad y^2 = 9\left(1-\frac{x^2}{16}\right)$$

$$\therefore \quad y^2 = \frac{9}{16}(16-x^2)$$

$$\therefore \quad y^2 = \frac{3}{4}\sqrt{16-x^2}$$

$$I = \int_0^4 y\ dx$$

$$I = \int_0^4 \frac{3}{4}\sqrt{16-x^2}\ dx$$

$$I = \frac{3}{4}\int_0^4 \sqrt{16-x^2}\ dx$$

$$I = \frac{3}{4}\left[\frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\left(\frac{x}{4}\right)\right]_0^4$$

$$I = \frac{3}{4}\left[\left(\frac{4}{2}(0) + 8\sin^{-1}(1)\right) - \left(0 + \sin^{-1}(0)\right)\right]$$

$$I = \frac{3}{4}\left(8\cdot\frac{\pi}{2}\right)$$

$I = 3\pi$

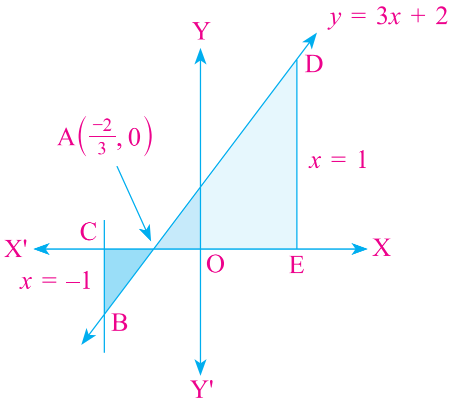
Now, $A = 4|I|$

$= 4|3\pi|$

$\therefore \quad A = 12\pi$ sq. units

6.

As shown in the fig., the line $y = 3x + 2$, meets X-axis at $\left(-\frac{2}{3}, 0\right)$ and its graph lie below X-axis for $x \in \left(-1, -\frac{2}{3}\right)$ and above X-axis for $x \in \left(-\frac{2}{3}, 1\right)$.



The required area

= Area of the region ACBA + Area of the region ADEA

$$= \left|\int_{-1}^{-\frac{2}{3}} (3x+2)\,dx\right| + \int_{-\frac{2}{3}}^{1} (3x+2)\,dx$$

$$= \left|\left(\frac{3}{2}x^2 + 2x\right)_{-1}^{-\frac{2}{3}}\right| + \left(\frac{3}{2}x^2 + 2x\right)_{-\frac{2}{3}}^{1}$$

$$= \left|\left(\frac{3}{2}\left(\frac{4}{9}\right) - \frac{4}{3}\right) - \left(\frac{3}{2}(1) + 2(-1)\right)\right| + \left(\frac{3}{2}(1) + 2(1)\right) - \left(\frac{3}{2}\left(\frac{4}{9}\right) + 2\left(-\frac{2}{3}\right)\right)$$

$$= \left|\frac{2}{3} - \frac{4}{3} - \frac{3}{2} + 2\right| + \frac{3}{2} + 2 - \frac{2}{3} + \frac{4}{3}$$

$$= \left|\frac{-2}{3} - \frac{3}{2} + 2\right| + \frac{3}{2} + 2 + \frac{2}{3}$$

$$= \left|\frac{-4 - 9 + 12}{6}\right| + \frac{9 + 12 + 4}{6}$$

$$= \frac{1}{6} + \frac{25}{6}$$

$$= \frac{26}{6}$$

$$= \frac{13}{3} \text{ sq. units.}$$

7.

$\sec^2 x \cdot \tan y\, dx + \sec^2 y \tan x\, dy = 0$

$\therefore \sec^2 x \tan y\, dx = -\sec^2 y \tan x\, dy$

$$\therefore \frac{\sec^2 y}{\tan y}\, dy = \frac{-\sec^2 x}{\tan x}\, dx$$

→ Integrate both the sides,

$$\int \frac{\sec^2 y}{\tan y}\, dy = -\int \frac{\sec^2 x}{\tan x}\, dx$$

$$\therefore \int \frac{\frac{d}{dy}(\tan y)}{\tan y}\, dy = -\int \frac{\frac{d}{dx}(\tan x)}{\tan x}\, dx$$

$$\therefore \log|\tan y| = -\log|\tan x| + \log|c|$$

$$\therefore \log|\tan y| = \log\left|\frac{c}{\tan x}\right|$$

$$\therefore \tan y = \frac{c}{\tan x}$$

$$\therefore \tan x \cdot \tan y = c;$$

Which is required general solution of given differential equation.

8.

Here,

$\overrightarrow{AB} = (1 - 2)\hat{i} + (-3 + 1)\hat{j} + (-5 - 1)\hat{k}$

$= -\hat{i} - 2\hat{j} - 6\hat{k}$

$AB = \sqrt{1 + 4 + 36} = \sqrt{41}$

$\overrightarrow{BC} = (3 - 1)\hat{i} + (-4 + 3)\hat{j} + (-4 + 5)\hat{k}$

$= 2\hat{i} - \hat{j} + \hat{k}$

$BC = \sqrt{4 + 1 + 1} = \sqrt{6}$

and

$\overrightarrow{CA} = (2 - 3)\hat{i} + (-1 + 4)\hat{j} + (1 + 4)\hat{k}$

$= -\hat{i} + 3\hat{j} + 5\hat{k}$

$CA = \sqrt{1 + 9 + 25} = \sqrt{35}$

Further, Note that

$|\overrightarrow{AB}|^2 = 41$

$= 6 + 35$

$= |\overrightarrow{BC}|^2 + |\overrightarrow{CA}|^2$

Hence, the triangle is a right angled triangle.

9.

Line L_1 : $\frac{x - 8}{3} = \frac{y + 19}{-16} = \frac{z - 10}{7}$

$\vec{r} = (8\hat{i} - 19\hat{j} + 10\hat{k}) + \lambda(3\hat{i} - 16\hat{j} + 7\hat{k})$

Direction of line $\vec{b_1} = 3\hat{i} - 16\hat{j} + 7\hat{k}$

Line L_2 : $\frac{x - 15}{3} = \frac{y - 29}{8} = \frac{z - 5}{-5}$

$\vec{r_2} = (15\hat{i} + 29\hat{j} + 5\hat{k}) + \lambda(3\hat{i} + 8\hat{j} - 5\hat{k})$

Direction of line $\vec{b_2} = 3\hat{i} + 8\hat{j} - 5\hat{k}$

$$\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$$

$= 24\hat{i} + 36\hat{j} + 72\hat{k}$

$= 12(2\hat{i} + 3\hat{j} + 6\hat{k})$

$\therefore$ Direction of given line $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

$A(\vec{a}) = \hat{i} + 2\hat{j} - 4\hat{k}$ line on the line

Vector equation of line :

$$\frac{x-x_1}{l_1} = \frac{y-y_1}{l_2} = \frac{z-z_1}{l_3}$$

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z+4}{6}$$

10.

We have

$\vec{a} = 5\hat{i} + 2\hat{j} - 4\hat{k}$ yLku $\vec{b} = 3\hat{i} - 2\hat{j} + 8\hat{k}$ Au.

Therefore, the vector equation of the line is

$\vec{r} = 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} - 2\hat{j} + 8\hat{k})$

Now, $\vec{r}$ is the position vector of any point P(x, y, z) on the line.

Therefore, $x\hat{i} + y\hat{j} + z\hat{k}$

$= 5\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(3\hat{i} + 2\hat{j} - 8\hat{k})$

$= (5 + 3\lambda)\hat{i} + (2 + 2\lambda)\hat{j} + (-4 - 8\lambda)\hat{k}$

Eliminating λ, we get

$$\frac{x-5}{3} = \frac{y-2}{2} = \frac{z+4}{-8}$$

Which is the equation of the line in cartesian form.

11.

P(E) = 0.6

P(F) = 0.3

$P(E \cap F) = 0.2$

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = \frac{0.2}{0.3} = \frac{2}{3}$$

$$P(F \mid E) = \frac{P(E \cap F)}{P(E)} = \frac{0.2}{0.6} = \frac{1}{3}$$

12.

If all the 36 elementary events of the experiment are considered to be equally likely, we have

$P(A) = \frac{18}{36} = \frac{1}{2}$ and

$P(B) = \frac{18}{36} = \frac{1}{2}$

Also, $P(A \cap B) = P$ (odd Number on both throws)

$$= \frac{9}{36} = \frac{1}{4}$$

Now, $P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

Cleary, $P(A \cap B) = P(A) \cdot P(B)$

Thus, A and B are independent events.

SECTION B

13.

Relation defined on R,

$S = \{(a, b) : a \le b^3\}$

For $a = \frac{1}{2}$, $\left(\frac{1}{2}, \frac{1}{2}\right) \notin S$ $\left(\because \frac{1}{2} \not< \frac{1}{8}\right)$

$\therefore$ $\left(\frac{1}{2}, \frac{1}{2}\right) \notin S$ $\therefore$ S is not reflexive.

Suppose, $(1, 5) \in S$

Then, $(5, 1) \notin S$ $(\because 5 \nleq 1)$

$\therefore$ S is not symmetric.

Suppose, $(a, b) \in S$ and $(b, c) \in S$

$\therefore$ $a \le b^3$ and $b \le c^3$

$\therefore$ $b^3 \le c^9$

Thus, $a \le b^3 \le c^9$

$\therefore$ $a \le c^9$

$\therefore$ $(a, c) \notin S$

$\therefore$ S is not transitive.

Hence, S is not reflexive, symmetric, transitive.

14.

$A^2 = A \cdot A$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+0+4 & 0+0+0 & 2+0+6 \\ 0+0+2 & 0+4+0 & 0+2+3 \\ 2+0+6 & 0+0+0 & 4+0+9 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$A^3 = A^2 \cdot A$

$$= \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 5+0+16 & 0+0+0 & 10+0+24 \\ 2+0+10 & 0+8+0 & 4+4+15 \\ 8+0+26 & 0+0+0 & 16+0+39 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

Now, L.H.S. = $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} + \begin{bmatrix} -30 & 0 & -48 \\ -12 & -24 & -30 \\ -48 & 0 & -78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 21-30+7+2 & 0+0+0+0 & 34-48+14+0 \\ 12-12+0+0 & 8-24+14+2 & 23-30+7+0 \\ 34-48+14+0 & 0+0+0+0 & 55-78+21+2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O = \text{R.H.S.}$$

15.

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Co-factor of the element yz $\quad A_{13} = (-1)^4 \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix}$

$= (1)(z - y)$

$= (z - y)$

Co-factor of the element zx $\quad A_{23} = (-1)^5 \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix}$

$= (-1)(z - x)$

$= x - z$

Co-factor of the element xy $\quad A_{33} = (-1)^6 \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix}$

$= (1)(y - x)$

$= y - x$

$\Delta = a_{13}A_{13} + a_{23}A_{23} + a_{33}A_{33}$

$= (yz)(z - y) + (zx)(x - z) + (xy)(y - x)$

$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$

$= z(x^2 - y^2) + z^2(y - x) + xy(y - x)$

$= z[(x - y)(x + y)] + z^2(y - x) + xy(y - x)$

$= (y - x)(-z(x + y) + z^2 + xy)$

$= (y - x)(-zx - yz + z^2 + xy)$

$= (y - x)(z(z - x) - y(z - x))$

$= (y - x)(z - x)(z - y)$

$= (x - y)(y - z)(z - x)$

16.

$y = (tan^{-1} x)^2$ Lkwt

Differentiate w.r.t. x,

$$\frac{dy}{dx} = 2tan^{-1} x \cdot \frac{1}{1+x^2}$$

$$\therefore y_1 = \frac{2\,tan^{-1} x}{1+x^2}$$

$\therefore (1 + x^2)\, y_1 = 2tan^{-1}x$

Differentiate again w.r.t. x,

$$\therefore (1 + x^2)\, y_2 + y_1 \cdot 2x = \frac{2}{(1+x^2)}$$

$\therefore (1 + x^2)^2\, y_2 + 2x(1 + x^2)\, y_1 = 2$

17.

$f(x) = 2x^3 - 24x + 107$

$\therefore f'(x) = 6x^2 - 24$

→ For finding maximum and minimum value,

$f'(x) = 0$

$\therefore 6x^2 - 24 = 0$

$\therefore 6x^2 = 24$

$\therefore x^2 = 4$

$\therefore x = \pm 2$

$\therefore x = 2 \in (1, 3)$ yLku $x = -2 \in (-3, -1)$

$f(2) = 2(2)^3 - 24(2) + 107$

$= 16 - 48 + 107$

$= 75$

→ $a = 1$, $b = 3$

$f(a) = f(1)$

$= 2(1)^3 - 24(1) + 107$

$= 2 - 24 + 107$

$= 85$

$f(b) = f(3)$

$= 2(3)^3 - 24(3) + 107$

$= 54 - 72 + 107 = 89$

Absolute maximum value = *max* {85, 89, 75}

= 89

→ $-2 \in (-3, -1)$

$f(-2) = 2(-2)^3 - 24(-2) + 107$

$= 139$

→ òu $a = -3$, $b = -1$

$f(a) = f(-3)$

$= 2(-3)^3 - 24(-3) + 107$

$= -54 + 72 + 107$

$= 125$

$f(b) = f(-1)$

$= 2(-1)^3 - 24(-1) + 107$

$= -2 + 24 + 107$

$= 129$

Absolute maximum value = *max* {139, 125, 129}

= 139

18.

$\vec{a} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

$\vec{b} = \lambda\hat{i} + 2\hat{j} + 3\hat{k}$

$\therefore \vec{a} + \vec{b} = (2 + \lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Now, $|\vec{a} + \vec{b}| = \sqrt{(2+\lambda)^2 + 36 + 4}$

$= \sqrt{4 + 4\lambda + \lambda^2 + 40}$

$= \sqrt{\lambda^2 + 4\lambda + 44}$

Unit vector in the direction of sum of vectors $\vec{a}$ and $\vec{b}$

$$= \frac{\vec{a} + \vec{b}}{|\vec{a} + \vec{b}|}$$

$$= \frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$$

Now, The scalar product of $\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}$ $\hat{i} + \hat{j} + \hat{k}$ with is equal to 1.

$$\therefore \left(\frac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{\lambda^2 + 4\lambda + 44}}\right) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1$$

$$\therefore \left(\frac{1}{\sqrt{\lambda^2 + 4\lambda + 44}}\right) (2 + \lambda + 6 - 2) = 1$$

$\therefore (\lambda + 6) = \sqrt{\lambda^2 + 4\lambda + 44}$

$\therefore (\lambda + 6)^2 = \lambda^2 + 4\lambda + 44$

$\therefore \lambda^2 + 12\lambda + 36 = \lambda^2 + 4\lambda + 44$

$\therefore 8\lambda = 8$

$\therefore \lambda = 1$

19.

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$$

L : $\vec{r} = (-\hat{i} - \hat{j} - \hat{k}) + \lambda(7\hat{i} - 6\hat{j} + \hat{k})$

and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

M : $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \mu(\hat{i} - 2\hat{j} + \hat{k})$

$\vec{a_1} = -\hat{i} - \hat{j} - \hat{k}$;

$\vec{b_1} = 7\hat{i} - 6\hat{j} + \hat{k}$

and $\vec{a_2} = 3\hat{i} + 5\hat{j} + 7\hat{k}$;

$\vec{b_2} = \hat{i} - 2\hat{j} + \hat{k}$

Now, $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 7 & -6 & 1 \\ 1 & -2 & 1 \end{vmatrix}$

$\vec{b_1} \times \vec{b_2} = -4\hat{i} - 6\hat{j} - 8\hat{k}$

$\neq \vec{0}$

$\therefore$ Lines are intersecting lines are skew lines.

Now, $|\vec{b_1} \times \vec{b_2}| = \sqrt{16 + 36 + 64}$

$= \sqrt{116}$

$= 2\sqrt{29}$

$\vec{a_2} - \vec{a_1} = 4\hat{i} + 6\hat{j} + 8\hat{k}$

$\therefore (\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})$

$= (4\hat{i} + 6\hat{j} + 8\hat{k}) \cdot (-4\hat{i} - 6\hat{j} - 8\hat{k})$

$= -16 - 36 - 64$

$= -116$

$\neq 0$

$\therefore$ Lines are skew line.

Shortest distance between two lines,

$$= \frac{|(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2})|}{\vec{b_1} \times \vec{b_2}}$$

$$= \frac{|-116|}{\sqrt{116}}$$

$= \sqrt{116}$

$= \sqrt{4 \times 29}$

$= 2\sqrt{29}$ unit

20.

$x + 2y \leq 8$

$3x + 2y \leq 12$

$x \geq 0$

$y \geq 0$

objective function $Z = -3x + 4y$

$x + 2y = 8$ ... (i)

x	0	8
y	4	0

$3x + 2y = 12$ ... (ii)

x	0	4
y	6	0

Solving equation (i) and (ii),

$\therefore 8 - x = 12 - 3x$

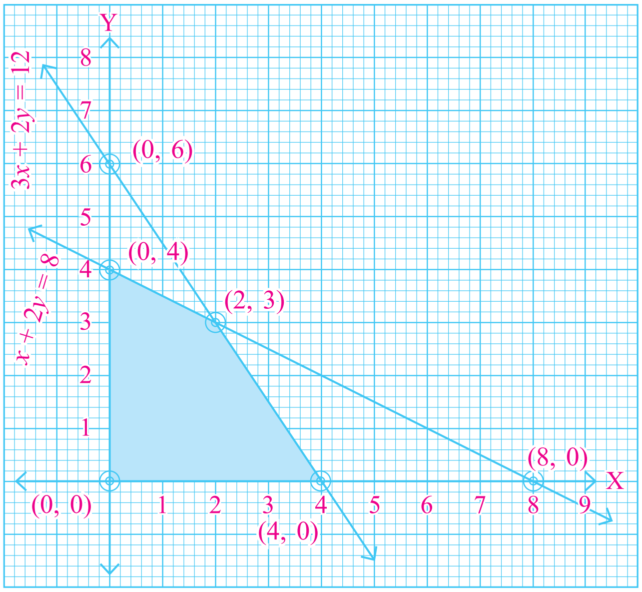
$\therefore 2x = 4$

$\therefore x = 2$

$\therefore y = 3$

(2, 3)

(0, 0)



The shaded region in fig. is feasible region determined by the system of constraints which is bounded. The co-ordinates of corner point (0, 0), (4, 0), (2, 3) and (0, 4).

Corner Point	Corresponding value of $Z = -3x + 4y$
(0, 4)	Z = 16
(4, 0)	Z = –12 ← Minimum
(2, 3)	Z = 6
(0, 0)	Z = 0

Thus, the Minimum value of 2 is –12 at point (4, 0).

21.

Let events B_1, B_2, B_3 be the following :

B_1 : the bolt is manufactured by machine A

B_2 : the bolt is manufactured by machine B

B_3 : the bolt is manufactured by machine C

Clearly, B_1, B_2, B_3 are mutually exclusive and exhaustive events and hence, they represent a partition of the sample space.

Let the event E be 'the bolt is defective'.

The event E occurs with B_1 or with B_2 or with B_3.

Given that,

$P(B_1) = 25\% = 0.25,$

$P(B_2) = 0.35$ and

$P(B_3) = 0.40$

Again

$P(E|B_1)$ = Probability that the bolt drawn is defective given that it is manufactured by machine

A = 5% = 0.05

Similarly, $P(E \mid B_2) = 0.04,$

$P(E \mid B_3) = 0.02$

Hence, by Bayes' Theorem, we have,

$$P(B_2 \mid E) = \frac{P(B_2)\cdot P(E \mid B_2)}{P(B_1)P(E \mid B_1)+P(B_2)P(E \mid B_2)+P(B_3)P(E \mid B_3)}$$

$$= \frac{0.35 \times 0.04}{0.25 \times 0.05 + 0.35 \times 0.04 + 0.40 \times 0.02}$$

$$= \frac{0.0140}{0.0345}$$

$$= \frac{28}{69}$$

SECTION C

22.

$$[x \quad -5 \quad -1]\begin{bmatrix}1 & 0 & 2\\0 & 2 & 1\\2 & 0 & 3\end{bmatrix}\begin{bmatrix}x\\4\\1\end{bmatrix} = O$$

$$\therefore \quad [x + 0 - 2 \quad 0 - 10 + 0 \quad 2x - 5 - 3]\begin{bmatrix}x\\4\\1\end{bmatrix} = O$$

$$\therefore \quad [x - 2 \quad -10 \quad 2x - 8]\begin{bmatrix}x\\4\\1\end{bmatrix} = O$$

$\therefore \quad [x(x - 2) + (-10)(4) + (2x - 8)(1)] = O$

$\therefore \quad [x^2 - 2x - 40 + 2x - 8] = O$

$\therefore \quad [x^2 - 48] = [0]$

$\therefore \quad x^2 - 48 = 0$

$\therefore \quad x^2 = 48$

$\therefore \quad x = \pm\sqrt{48} = \pm 4\sqrt{3}$

23. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 6,$

$\frac{1}{y} + \frac{3}{z} = 11,$

$\frac{1}{x} - \frac{2}{y} + \frac{1}{z} = 0$

The equation can be written as matrix form,

$$\begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}\begin{bmatrix}\frac{1}{x}\\\frac{1}{y}\\\frac{1}{z}\end{bmatrix} = \begin{bmatrix}6\\11\\0\end{bmatrix}$$

$$\therefore \quad A = \begin{bmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{bmatrix}, X = \begin{bmatrix}\frac{1}{x}\\\frac{1}{y}\\\frac{1}{z}\end{bmatrix}, B = \begin{bmatrix}6\\11\\0\end{bmatrix}$$

$$|A| = \begin{vmatrix}1 & 1 & 1\\0 & 1 & 3\\1 & -2 & 1\end{vmatrix}$$

$= 1(1 + 6) - 1(0 - 3) + 1(0 - 1)$

$= 7 + 3 - 1$

$= 9 \neq 0$

$\therefore \quad A^{-1}$ exists.

$$\text{adj A} = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|} \text{adj A}$$

$$= \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$\therefore \quad AX = B$

$\therefore \quad A^{-1}AX = A^{-1}B$

$\therefore \quad IX = A^{-1}B$

$\therefore \quad X = A^{-1}B$

$$= \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 42 & -33 & +0 \\ 18 & +0 & -0 \\ -6 & +33 & +0 \end{bmatrix}$$

$$= \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

$\therefore$ **Solution :** $\frac{1}{x} = 1, \ \frac{1}{y} = 2, \ \frac{1}{z} = 3$

$\therefore x = 1, y = \frac{1}{z}, z = \frac{1}{3}$

24.

$x = a(cost + tsint)$

$\therefore \frac{dx}{dt} = a(-sint + tcost + sint)$

$\therefore \frac{dx}{dt} = atcost$

Now, $y = a(sint - tcost)$

$\therefore \frac{dy}{dt} = a[cost + tsint - cost]$

$\therefore \frac{dy}{dt} = atsint$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{at\sin t}{at\cos t} = tant$$

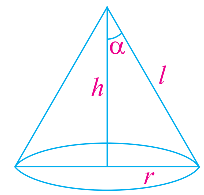
Now, differentiate again w.r.t. x,

$$\frac{d^2y}{dx^2} = \frac{d}{dt}\left(\frac{dy}{dx}\right) \cdot \frac{dt}{dx} = \frac{d}{dt}(\tan t) \cdot \frac{dt}{dx}$$

$$\frac{d^2y}{dx^2} = sec^2t \ \frac{dt}{dx} = \frac{sec^2 t}{at\cos t}$$

$$\frac{d^2y}{dx^2} = \frac{sec^3 t}{at}; \ 0 < t < \frac{\pi}{2}$$

25.



Suppose, radius of cone is r height is h and slant height is l.

$\therefore \ l^2 = h^2 + r^2$ .... (1)

Suppose, semi-vertical angle is α.

$\therefore \ \tan\alpha = \frac{r}{h}$

$\therefore \quad \alpha = \tan^{-1}\left(\frac{r}{h}\right)$

$\rightarrow$ Volume of cone $V = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi(l^2 - h^2)h$

($\because$ From equation (1))

$\therefore \quad f(h) = \frac{\pi}{3}(l^2h - h^3)$

$f'(h) = \frac{\pi}{3}(l^2 - 3h^2)$

$\therefore \quad f''(h) = \frac{\pi}{3}(-6h)$

$\therefore \quad f''(h) = -2\pi h < 0$

$\therefore$ f has minimum value

$\rightarrow$ For finding maximum volume of cone,

$f'(h) = 0$

$\therefore \quad \frac{\pi}{3}(l^2 - 3h^2) = 0$

$\therefore \quad l^2 - 3h^2 = 0$

$\therefore \quad h^2 + r^2 - 3h^2 = 0$

$\therefore \quad r^2 - 2h^2 = 0$

$\therefore \quad r^2 = 2h^2$

$\therefore \quad r = \sqrt{2}\,h$

$\therefore \quad \frac{r}{h} = \sqrt{2}$

$\therefore$ Semi-vertical angle $= \tan^{-1}\left(\frac{r}{h}\right)$

$= \tan^{-1}(\sqrt{2})$

26.

$$I = \int_0^{\frac{\pi}{4}} \log(1 + \tan x)\,dx \qquad \text{... (1)}$$

By property (6), $x = \frac{\pi}{4} - x$

$$I = \int_0^{\frac{\pi}{4}} \log\left(1 + \tan\left(\frac{\pi}{4} - x\right)\right)dx$$

$$= \int_0^{\frac{\pi}{4}} \log\left[1 + \frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x}\right]dx$$

$$= \int_0^{\frac{\pi}{4}} log\left(1+\frac{1-tan\,x}{1+tan\,x}\right) dx$$

$$I = \int_0^{\frac{\pi}{4}} log\left(\frac{1+tan\,x+1-tan\,x}{1+tan\,x}\right) dx$$

$$= \int_0^{\frac{\pi}{4}} log\left(\frac{2}{1+tan\,x}\right) dx$$

$$= \int_0^{\frac{\pi}{4}} (log(2) - log(1 + tan\,x))\, dx$$

$$I = log\,2 \int_0^{\frac{\pi}{4}} 1\,dx - \int_0^{\frac{\pi}{4}} log(1 + tan\,x)\, dx$$

$$I = log\,2\,[x]_0^{\frac{\pi}{4}} - I \qquad (\because \text{ From equation (1)})$$

$$2I = log\,2\left(\frac{\pi}{4}-0\right)$$

$$\therefore\ I = \frac{\pi}{8}\, log\,2$$

27.

The given differential equation can be written as :

$$\frac{dx}{dy} = \frac{2x(e)^{\frac{x}{y}} - y}{2y(e)^{\frac{x}{y}}} \qquad \text{... (1)}$$

Let,

$$F(x, y) = \frac{2x(e)^{\frac{x}{y}} - y}{2y(e)^{\frac{x}{y}}}$$

$$\therefore\ F(\lambda x, \lambda y) = \frac{\left(2x(e)^{\frac{x}{y}} - y\right)}{\left(2y(e)^{\frac{x}{y}}\right)}$$

$$= \lambda^0\, F(x, y)$$

Thus, $F(x, y)$ is a homogeneous function of degree zero. Therefore, the given differential equation is a homogeneous differential equation.

To solve it, we make the substitution

$$x = vy \qquad \text{... (2)}$$

Differentiating equation (2) with respect to y, we get

$$\frac{dx}{dy} = v + y\frac{dv}{dy}$$

Substituting the value of x and $\frac{dx}{dy}$ in equation (1), we get,

$$v + y\frac{dv}{dy} = \frac{2\,ve^v - 1}{2e^v}$$

$$\therefore\quad y\frac{dv}{dy} = \frac{2\,ve^v - 1}{2e^v} - v$$

$$\therefore\quad y\frac{dv}{dy} = -\frac{1}{2e^v}$$

$$\therefore\quad 2e^v\,dv = -\frac{dy}{y}$$

$$\therefore\quad \int 2e^v\,dv = -\int\frac{dy}{y}$$

$$\therefore\quad 2e^v = -\,log\,|y| + c$$

and replacing v by $\frac{x}{y}$, we get,

$$2(e)^{\frac{x}{y}} + log|y| + c \qquad \text{.... (3)}$$

Substituting $x = 0$ and $y = 1$ in equation (3), we get

$$2e^0 + log\,|1| = c \Rightarrow c = 2$$

Substituting the value of c in equation (3), we get

$$2(e)^{\frac{x}{y}} + log|y| = 2$$

which is the particular solution of the given differential equation.